

QUOTING EXPERIMENTAL INFORMATION

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ABSTRACT

There was a question raised at this meeting as to the best way to present the results of experiments measuring anisotropies in the CMB. Here we will make some simple comments about the 3 main competing methods and some suggestions. In particular we will give an over-simplified but hopefully useful method of comparing GACF numbers with other power spectra.

Subject headings: cosmic background radiation — cosmology: theories and observations — GACF's: justified

1. Introduction

There are a number of ways of quoting the level of anisotropy measured by a CMB experiment. Some of the most common are: the rms temperature measured by the experiment T_{rms} ; the amplitude and correlation angle of a Gaussian AutoCorrelation Function (GACF) C_0 and θ_c ; the amplitude of a 'flat' or Harrison-Zel'dovich power spectrum; and the amplitude of a 'standard' spectrum such as CDM.

The rms temperature measured by the experiment is the quantity most directly related to the data, but also the least informative when it comes to comparing different experiments, since it depends on the window function and calibration procedures etc. of the experiments. It also does not include the sample¹ and cosmic variance associated with the measurement. All the other methods choose to quote the amplitude of fluctuations by assuming the power spectrum has a certain form and fitting for the amplitude. This quantity is independent of the window function normalization, calibration etc., and if the amplitude really is fit properly to the data, cosmic and sample variance are included in the result automatically. As long as the *same* form is assumed for *all* experiments, such numbers can be safely compared from experiment to experiment and with theories.

Since the ability to compare experiments is certainly something we would all like to have, it seems then that fitting a power spectrum is the way to go. One suggestion (Scott Dodelson, this meeting), is to define a 'standard' model such as CDM with a fixed Ω_B and h , and to quote the amplitude of such a spectrum. This has the advantage that, to the extent that such a model correctly describes the fluctuations on the sky, the amplitudes so obtained can be trivially compared from

experiment to experiment: they should be all the same. The drawbacks of this particular form for the power spectrum are that it involves an enormous amount of theoretical prejudice at the level even of *quoting* the experimental measurements, and that the ‘standard’ power spectrum has a large amount of structure in it. Should the model turn out to be something other than ‘standard CDM’ or if Ω_B or h turn out to be different than was assumed, this structure has to be deconvolved from the measurements before a new power spectrum is used. We note in passing that comparison of C_ℓ ’s from different codes has not been done in detail to let us understand the reproducibility of the curves.

At the other extreme are the flat spectrum and the GACF. These are not at all well motivated as models of our sky, but they are extremely simple power spectra. While measurements on different scales are not expected to give the *same* amplitude (Doppler peaks give a larger amplitude for experiments on degree scales, for example), it is not difficult to remember a few numbers at a range of scales for comparison, and they have even been tabulated for CDM².

So what are the merits and disadvantages of GACF’s and flat spectra? Historically experimental results (upper limits until recently) have been quoted in terms of the amplitude of a ‘most sensitive’ GACF. This method pre-dates the emergence of window functions into the popular consciousness and gives an alternate view of the ‘scale’ at which the experiment is sensitive and the degree of the sensitivity^{3,4}. As discussed in references 2 and 5 the GACF analysis is straightforward to understand in terms of window functions, and a conversion from GACF numbers to ‘flat spectrum’ numbers is easy to do^{2,6,7,8}. Note that the advantage of having a readily understandable scale in your fit (θ_c) is also a *disadvantage* when it comes to comparing experiments, since values of C_0 for different (arbitrary) θ_c cannot be directly compared. The flat spectrum has no such problem since it is featureless and easily understandable, but does not directly contain the information on the window function that the GACF encodes.

Here we would like to make some basic remarks about the relation of the GACF to the window function and about converting from GACF’s to flat spectra and back. The details of such a conversion have been treated before, but we would like to present a simple (and not necessarily very accurate!) method which hopefully illustrates the general idea.

2. To and from the GACF

Since our aim here is simplicity rather than rigour, we are going to assume that all CMB experiments on small scales are two-beam square-wave chop experiments, with peak-to-peak chop α and (gaussian) beam width σ . Under this approximation the window function is of the well known form

$$\begin{aligned} W_\ell &= 2 [1 - P_\ell(\cos \alpha)] e^{-\ell(\ell+1)\sigma^2} \\ &\simeq \frac{1}{2}(\ell\alpha)^2 e^{-(\ell\sigma)^2}, \end{aligned} \tag{1}$$

where in the second line we have made the approximation $\alpha \ll 1$, or more cor-

rectly that $\ell\alpha < 1$ for the ℓ range of interest. This is not a tremendously good approximation, but brings out the essential features of what is going on.

Now in the language of power spectra and window functions, the most important quantity measured by an experiment is the power through the window, which is given by

$$\text{Power} = \frac{1}{4\pi} \sum_{\ell=2}^{\infty} (2\ell+1) C_{\ell} W_{\ell}. \quad (2)$$

As experiments become more sensitive, other quantities (e.g. correlations) are going to play an important role⁹, but for now the power is what most experiments are sensitive to⁵. For experiments on small or intermediate angular scales we can approximate the sum as an integral and replace $(2\ell+1) \rightarrow 2\ell$. If the window function drops off at small ℓ we are safe in extending the lower limit of the integration to $\ell = 0$. The quantity normally plotted with the window function is $\ell(\ell+1)C_{\ell} \approx \ell^2 C_{\ell}$ in terms of which

$$\text{Power} \approx \frac{2}{4\pi} \int_{-\infty}^{\infty} \ell^2 C_{\ell} W_{\ell} d \log \ell, \quad (3)$$

which explains the logarithmic ℓ -axis normally used in such plots.

What do the two ‘competing’ power spectra look like? The flat spectrum is extremely simple

$$\ell(\ell+1)C_{\ell} = \frac{24\pi}{5} Q_{\text{flat}}^2 = \text{constant}. \quad (4)$$

The factor of $24\pi/5$ is there simply to soak up the constant in Eq. (2) so that Q_{flat}^2 is the coefficient of W_2 á la COBE. The GACF power spectrum on the other hand is

$$\ell(\ell+1)C_{\ell} = 2\pi C_0 \ell^2 \theta_c^2 e^{-1/2 \ell^2 \theta_c^2}. \quad (5)$$

Notice now the similarity between $\ell(\ell+1)C_{\ell}$ for the GACF and the expression for the window function of Eq. (1). Keeping in mind that the power measured by the experiment is the fixed quantity defined by the data, it will come as no surprise that the θ_c which gives the lowest C_0 is $\theta_c = \sqrt{2}\sigma \simeq 0.6 \times \text{FWHM}$. This is the value of θ_c which maximizes the power through the window function for a GACF [using Eq. (1) and Eq. (5) in Eq. (3)]. All that varying θ_c in the fit is doing is matching the power spectrum to the window function of the experiment, and this is another way of encoding the information about which ℓ ’s the experiment measures. In the *highly simplified* approximation for window functions given by Eq. (1), the correlation angle of the experiment is set by the beam size, which also sets the peak of the window function, which is $\ell_0 = 1/\sigma$. In practice this last relation is very poor (see Table 1), since getting the peak correct requires accurately modelling the left hand rise of W_{ℓ} well for ‘large’ $\ell\alpha$ unless, $\sigma \gg \alpha$. This needs a detailed consideration of each experiment¹⁰.

We can do some easy integrals to see how the GACF-to-flat spectrum conversion works in our over-simplified but hopefully instructive approximation. The power through the window of the flat spectrum is proportional to the area under the

window function vs. $\log \ell$, or

$$\begin{aligned} \text{Power} &= \frac{6}{5} Q_{\text{flat}}^2 \int_0^\infty d\ell^2 \ell^{-2} W_\ell \\ &= \frac{6}{5} Q_{\text{flat}}^2 \frac{\alpha^2}{\sigma^2}. \end{aligned} \quad (6)$$

While the power through the window for a GACF is

$$\begin{aligned} \text{Power} &= \frac{1}{2} C_0 \theta_c^2 \int_0^\infty d\ell^2 e^{-1/2 \ell^2 \theta_c^2} W_\ell \\ &= C_0 \frac{\alpha^2}{4\sigma^2}, \end{aligned} \quad (7)$$

where in the second line we have evaluated the integral for $\theta_c = \sqrt{2}\sigma$. Relating these two we see that $Q_{\text{flat}} \simeq 0.46 C_0^{1/2}$, which compares well with numbers obtained using the accurate W_ℓ 's and the θ_c 's quoted for each experiment (see Table 1).

Table 1: Summary of the peak of the window function and the $C_0^{1/2}$ to Q_{flat} conversion factor for current CMB experiments on small and intermediate scales. Note that the peak of the window function does not correspond very well to $1/\sigma$, showing that $\sigma \gg \alpha$ is not a good approximation for most experiments. However, the $C_0^{1/2}$ to Q_{flat} conversion is close to 0.46–0.50, for many experiments, as predicted by simple calculation.

Experiment	ℓ_0	$1/\sigma$	θ_c	$Q/C_0^{1/2}$
Tenerife	20	25	4°0	0.50
SP91	66	96	1°5	0.44
SK93	71	93	1°2	0.49
Python	73	180	1°0	0.47
ARGO	107	156	0°5	0.42
MAX	158	270	0°5	0.45
MSAM2	143	289	0°5	0.44
MSAM3	249	289	0°3	0.50

We can do a little better for some experiments by noticing that they perform a double-difference (‘triple-beam’) rather than a single difference (‘double-beam’) measurement. This means that the window function is better represented by the functional form $W_\ell \propto \ell^4 e^{-\ell(\ell+1)\sigma^2}$ than by the lower order expression in Eq. (1). This changes the peak to $\ell_0 = \sqrt{2}/\sigma$, and the power of a GACF through this window peaks at $\theta_c = \sigma$. The comparison of a GACF and a flat spectrum then leads to $Q_{\text{flat}}/C_0^{1/2} = 2\sqrt{5}/9 \simeq 0.50$. We would expect Tenerife, SK93, Python and MSAM3 to be better characterized by this double-difference approximation. A glance at Table 1 shows that the $Q/C_0^{1/2}$ ratios are indeed higher.

3. Conclusions

In reality the approximations made above are not quite adequate for accurately

converting from GACF numbers to other measures of the power, but the procedure has the virtue of being very straightforward: one calculates the power in Eq. (2) for whatever power spectrum was used in the quoted answer. Assuming that this is the important information that the experiment provides, you adjust the normalization of your favourite power spectrum to match this number. Note that as long as you use the same window function in both calculations, the W_ℓ normalization doesn't matter. This is a boon because getting the shape of the window function is usually a lot easier than determining the height! Scaling from numbers quoted for power spectrum fits has the advantage that *all* of the experimental and theoretical errors will be included in such a fit.

Hopefully working through this over-simplified example has given some insight into the relation between the well loved GACF method and the ℓ -space measures preferred by theorists.

The bottom line: If you want a quick and dirty C_0 to Q conversion, then a factor of two is good at the 10% level for all experiments to date.

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